My hands are mostly fine now 🌛 but I'll steal UW slides 1 more time

Motivation

- SQL is a declarative language: we say what, we don't say how
- The query optimizer needs to convert the query into some intermediate language that can be both optimized, and executed
- That language is Relational Algebra

The Five Basic Relational Operators

- 1. Selection $\sigma_{\text{condition}}(S)$
- 2. Projection $\Pi_{\text{attrs}}(S)$
- 3. Join $\mathbb{R} \Join_{\theta} \mathbb{S} = \sigma_{\theta}(\mathbb{R} \times \mathbb{S})$
- 4. Union ∪
- 5. Set difference –

Rename ρ

Let's discuss them one by one

$\sigma_{\text{condition}}(T)$

Returns those tuples in T that satisfy the condition:

SELECT *

FROM T

WHERE condition;

$\sigma_{\text{condition}}(T)$

Returns those tuples in T that satisfy the condition:

SELECT *

FROM T

WHERE condition;

 $\sigma_{salary \ge 55000}(Payroll) =$

UserID	Name	Job	Salary
123	Jack	ТА	50000
345	Allison	ТА	60000
567	Magda	Prof	90000
789	Dan	Prof	100000

1. Selection

 $\sigma_{\text{condition}}(T)$

UserID	Name	Job	Salary
345	Allison	TA	60000
567	Magda	Prof	90000
789	Dan	Prof	100000

Returns those tuples in T that satisfy the condition:

SELECT *

FROM T

WHERE condition;

 $\sigma_{salary \ge 55000}(Payroll) =$

UserID	Name	Job	Salary
123	Jack	ТА	50000
345	Allison	ТА	60000
567	Magda	Prof	90000
789	Dan	Prof	100000

$\sigma_{\text{condition}}(T)$

Returns those tuples in T that satisfy the condition:

SELECT *

FROM T

WHERE condition;

 $\sigma_{\text{salary} \ge 55000 \text{ and } Job='TA'}(Payroll) =$

UserID	Name	Job	Salary
123	Jack	TA	50000
345	Allison	ТА	60000
567	Magda	Prof	90000
789	Dan	Prof	100000

1. Selection

 $\sigma_{\text{condition}}(T)$

UserID	Name	Job	Salary
345	Allison	ТА	60000

Returns those tuples in T

that satisfy the condition:

SELECT *

FROM T

WHERE condition;

$\sigma_{\text{salary} \ge 55000 \text{ and } \text{Job}='_{\text{TA}}}$, (Payroll) =

UserID	Name	Job	Salary
123	Jack	ТА	50000
345	Allison	ТА	60000
567	Magda	Prof	90000
789	Dan	Prof	100000

$\Pi_{\text{attrs}}(\mathbf{T})$

Returns all tuples in T keeping only the attributes in the subscript:

SELECT attrs
FROM T;

$\Pi_{\text{attrs}}(\mathbf{T})$

Returns all tuples in T keeping only the attributes in the subscript:

 $\Pi_{\text{Name,Salary}}(\text{Payroll}) =$

SELECT attrs **FROM** T;

Payroll			
UserID	Name	Job	Salary
123	Jack	TA	50000
345	Allison	TA	60000
567	Magda	Prof	90000
789	Dan	Prof	100000

П	(ፐ)
¹¹ attrs	

Name	Salary
Jack	50000
Allison	60000
Magda	90000
Dan	100000

Returns all tuples in T keeping only the attributes in the subscript:

SELECT attrs
FROM T;

 $\Pi_{\text{Name,Salary}}(\text{Payroll}) =$

Payroll			
UserID	Name	Job	Salary
123	Jack	TA	50000
345	Allison	TA	60000
567	Magda	Prof	90000
789	Dan	Prof	100000

$\Pi_{\text{attrs}}(\mathbf{T})$

Returns all tuples in T keeping only the attributes in the subscript:

 $\Pi_{Job}(Payroll) =$

SELECT attrs
FROM T;

Payroll			
UserID	Name	Job	Salary
123	Jack	TA	50000
345	Allison	TA	60000
567	Magda	Prof	90000
789	Dan	Prof	100000

2. Projection

 $\Pi_{\text{attrs}}(\mathbf{T})$

Job TA TA Prof Prof

Returns all tuples in T keeping only the attributes in the subscript:

 $\Pi_{Job}(Payroll) =$



SELEC	CT	attrs
FROM	Т;	

Payroll						
UserID	Name	Job	Salary			
123	Jack	TA	50000			
345	Allison	TA	60000			
567	Magda	Prof	90000			
789	Dan	Prof	100000			

2. Projection

п	(7	[]	
attrs		J	

Job
TA
TA
Prof

Prof

RA can be defined using bag semantics or set semantics. We always need to say which one we mean.

Returns all tuples in T keeping only the attributes in the subscript:

 $\Pi_{Job}(Payroll) =$

SELEC	CT	attrs
FROM	Τ;	

Payroll						
UserID	Name	Job	Salary			
123	Jack	TA	50000			
345	Allison	TA	60000			
567	Magda	Prof	90000			
789	Dan	Prof	100000			

Job

TA

Prof

3. Join

$S \Join_{\theta} T$

Join S and T using condition $\boldsymbol{\theta}$



3. Join

$S\Join_\theta T$

Join S and T using condition θ

Payroll $\bowtie_{\text{UserID}=\text{UserID}}$ Regist =



UserID	Name	Job	Salary	Regist	
123	Jack	TA	50000	UserID	Car
345	Allison	TA	60000	123	Charger
567	Magda	Prof	90000	567	Civic
789	Dan	Prof	100000	567	Pinto

3. Join

S ⋈_θ T

UserID	Name	Job	Salary	UserID	Car
123	Jack	TA	50000	123	Charger
567	Magda	Prof	90000	567	Civic
567	Magda	Prof	90000	567	Pinto

Join S and T using condition $\boldsymbol{\theta}$





Payrol

UserID	Name	Job	Salary	Regist	
123	Jack	TA	50000	UserID	Car
345	Allison	TA	60000	123	Charger
567	Magda	Prof	90000	567	Civic
789	Dan	Prof	100000	567	Pinto

- Eq-join: Payroll ⋈_{UserID=UserID} Regist
- Theta-join: Payroll ⋈_{UserID<UserID} Regist
- Cartesian product: Payroll × Regist
- Natural Join: Payroll 🛛 Regist

Many Variants of Join

■ Eq-join: Payroll ⋈_{UserID=UserID} Regist

- Theta-join: Payroll ⋈_{UserID} Regist
- Cartesian product: Payroll × Regist
- Natural Join: Payroll ⋈ Regist

Only =

Any condition

Many Variants of Join

■ Eq-join: Payroll ⋈_{UserID=UserID} Regist

■ Theta-join: Payroll ⋈_{UserID} Regist

Cartesian product: Payroll × Regist

■ Natural Join: Payroll 🛛 Regist

Only =

Any condition

Next

 $S \times T$

Cross product of S and T



$S \times T$

SELECT

FROM S,T

Cross product of S and T

*

$Payroll \times Regist =$

UserID	Name	Job	Salary	Regist	
123	Jack	TA	50000	UserID	Car
345	Allison	TA	60000	123	Charger
567	Magda	Prof	90000	567	Civic
789	Dan	Prof	100000	567	Pinto



 $Payroll \times Regist =$

J

SELECT * FROM S,T

UserID	Name	Job	Salary	Regist	
123	Jack	TA	50000	UserID	Car
345	Allison	TA	60000	123	Charger
567	Magda	Prof	90000	567	Civic
789	Dan	Prof	100000	567	Pinto

 $S \times T$

Cross product of S and T



Join = cartesian product + selection

 $R \bowtie_{\theta} S = \sigma_{\theta}(R \times S)$

S ⋈ T

Join S, T on common attributes, retain only one copy of those attributes

S ⋈ T

Join S, T on common attributes, retain only one copy of those attributes

Payroll \bowtie Regist =

UserID	Name	Job	Salary	Regist	
123	Jack	TA	50000	UserID	Car
345	Allison	TA	60000	123	Charger
567	Magda	Prof	90000	567	Civic
789	Dan	Prof	100000	567	Pinto

S ⋈ T

Join S, T on common attributes, retain only one copy of those attributes



UserID	Name	Job	Salary	Regist	
123	Jack	TA	50000	UserID	Car
345	Allison	TA	60000	123	Charger
567	Magda	Prof	90000	567	Civic
789	Dan	Prof	100000	567	Pinto

What do these natural joins output? $R(A,B) \bowtie S(B,C)$

 $\blacksquare R(A,B) \bowtie S(C,D)$

What do these natural joins output? $R(A,B) \bowtie S(B,C)$



$$\blacksquare R(A,B) \bowtie S(A,B)$$

What do these natural joins output?

■ $R(A,B) \bowtie S(B,C)$ eqjoin on attribute B (5 tuples)



 $\blacksquare R(A,B) \bowtie S(C,D)$

What do these natural joins output?

■ $R(A,B) \bowtie S(B,C)$ eqjoin on attribute B (5 tuples)



 $\blacksquare R(A,B) \bowtie S(C,D)$



What do these natural joins output?

■ $R(A,B) \bowtie S(B,C)$ eqjoin on attribute B (5 tuples)



■ $R(A,B) \bowtie S(C,D)$ cross product (12 tuples)



What do these natural joins output?

■ $R(A,B) \bowtie S(B,C)$ eqjoin on attribute B (5 tuples)



 $R(A,B) \bowtie S(C,D) \\ cross product (12 tuples)$





What do these natural joins output?

■ $R(A,B) \bowtie S(B,C)$ eqjoin on attribute B (5 tuples)



■ $R(A,B) \bowtie S(C,D)$ cross product (12 tuples)

intersection (2 tuples)



Even More Joins

■ Inner join 🖂

- Eq-join, theta-join, cross product, natural join
- Outer join
 - Left outer join ⋈
 - Right outer join ⋈
 - Full outer join ⋈
- Semi join ĸ

4. Union

S U T

The union of S and T


4. Union

$S \cup T$

The union of S and T

Regist \cup Bicycle =

S UNION T;

Regist		Bicycle
UserID	Model	UserID
123	Charger	345
567	Civic	567
567	Pinto	

UserID	Model
345	Schwinn
567	Sirrus

4. Union

$S \cup T$

The union of S and T



S UNION T;



4. Union

S U T

The union of S and T

Τ;

UNION

S



5. Difference

S - T

The set difference of S and T



5. Difference

S - T

The set difference of S and T

Regist - Bicycle =

S EXCEPT T;



5. Difference



 $\rho_{attrs}(T)$

Rename attributes

 $\rho_{attrs}(T)$

Rename attributes

$\rho_{\text{UserID,Model}}(\text{Regist}) =$

Regist

SELECT	a1	as	al',	
	a2	as	a2′,	
	• • •	•		
FROM T;	•			

UserID	Car
123	Charger
567	Civic
567	Pinto

		UserID	Model
		123	Charger
$\rho_{attrs'}(I)$		567	Civic
		567	Pinto
			egist) =
Rename attributes	PUSETID,MC	Regist	-910()
Rename attributes	P USETID,MC	Regist UserID	Car
Rename attributes	P USETID,MO	Regist UserID 123	Car Charger
Rename attributes	P USETID,MI	Regist UserID 123 567	Car Charger Civic
Rename attributes SELECT al as al', a2 as a2', 	P USETID,MI	Regist UserID 123 567 567	Car Charger Civic Pinto

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		UserID	Model
		123	Charger
$\rho_{attrs}(I)$		567	Civic
		567	Pinto
	0	(D	ordict) -
Rename attributes	ρ _{UserID,Mo}	Regist	
Rename attributes	PUserID,Mo	Regist	Car
Rename attributes	PUserID,Mo	Regist UserID 123	Car Charger
Rename attributes SELECT al as al', a2 as a2',	PUserID,Mo	Regist UserID 123 567	Car Charger Civic
Rename attributes SELECT a1 as a1', a2 as a2',	PUserID,Mo	Regist UserID 123 567 567	Car Charger Civic Pinto

Corrected union:

 $\rho_{\text{UserID,Model}}(\text{Regist}) \cup \text{Bicycle}$

The Five Basic Relational Operators

- 1. Selection $\sigma_{\text{condition}}(S)$
- 2. Projection $\Pi_{attrs}(S)$
- 3. Join $R \Join_{\theta} S = \sigma_{\theta}(R \times S)$
- 4. Union ∪
- 5. Set difference –

Rename ρ

Which operators are monotone?

The Five Basic Relational Operators



Which operators are monotone?

Query Plans



Payroll

UserID	Name	Job	Salary	Regist	
123	Jack	TA	50000	UserID	Car
345	Allison	TA	60000	123	Charger
567	Magda	Prof	90000	567	Civic
789	Dan	Prof	100000	567	Pinto

```
SELECT P.Name
FROM Payroll P, Regist R
WHERE P.UserID = R.UserID
and P.Job = 'TA';
```

 $\Pi_{\text{Name}}(\sigma_{\text{Job}='\text{TA}'}(\text{Payroll} \bowtie \text{Regist}))$

Payroll

UserID	Name	Job	Salary	Regist	
123	Jack	TA	50000	UserID	Car
345	Allison	TA	60000	123	Charger
567	Magda	Prof	90000	567	Civic
789	Dan	Prof	100000	567	Pinto





789

Dan

Prof

100000

567

Pinto

Query Plan: Attribute Names

Managing attribute names correctly is tedious

Better: use aliases, much like in SQL



```
SELECT P.Name
FROM Payroll P, Regist R
WHERE P.UserID = R.UserID
and P.Job = 'TA';
```

We say what we want, not how to get it



We say what we want, not how to get it







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Relational Algebra

60



Discussion

 Database system converts a SQL query to a Relational Algebra Plan

- Database system converts a SQL query to a Relational Algebra Plan
- Then it optimizes the plan by exploring equivalent plans, using simple algebraic identities:

$$R \bowtie S = S \bowtie R$$

$$R \bowtie (S \bowtie T) = (R \bowtie S) \bowtie T$$

$$\sigma_{\theta}(R \bowtie S) = \sigma_{\theta}(R) \bowtie S$$

... many others*

SQL to RA



SQL to RA



 $\hfill \hfill \hfill$

• Group-by aggregate γ_{attr1,attr2,...,agg1,...}

$\delta(T)$

Eliminates duplicates from the bag T

SELECT DISTINCT *

FROM T;

$\delta(T)$

Eliminates duplicates from the bag T

SELECT DISTINCT *
FROM T;

 $\delta(\mathbf{R}) =$



 $\delta(T)$

Eliminates duplicates from the bag T

SELECT DISTINCT *
FROM T;

AB110210220





 $\gamma_{attr1,attr2,...,agg1,...}(T)$

Group-by, then aggregate

SELECT attr1,...,agg1,...
FROM T
GROUP BY attr1,...;

 $\gamma_{attr1,attr2,...,a\underline{g}\underline{g}1,...}(\Gamma)$

Group-by, then aggregate

 $\gamma_{\text{Job}, avg(\text{Salary}) \rightarrow S}(\text{Payroll}) =$

SELECT attr1,...,agg1,...
FROM T
GROUP BY attr1,...;

Payroll			
UserID	Name	Job	Salary
123	Jack	TA	50000
345	Allison	TA	60000
567	Magda	Prof	90000
789	Dan	Prof	100000

 $\gamma_{attr1,attr2,...,a\underline{g}\underline{g}1,...}(\Gamma)$



Group-by, then aggregate

 $\gamma_{Job,avg(Salary) \rightarrow S}(Payroll) = -$

SELECT attr1,...,agg1,...
FROM T
GROUP BY attr1,...;

Payro	DII			
User	ID Nam	e Job	Salary	
123	Jack	TA	50000	
345	Allisc	on TA	60000	
567	Mago	la Prof	90000	
789	Dan	Prof	100000	

No need for a HAVING operator!

Find all jobs where the average salary of employees earning over 55000 is < 70000

UserID	Name	Job	Salary
123	Jack	TA	50000
345	Allison	TA	60000
567	Magda	Prof	90000
789	Dan	Prof	100000

Payroll

No need for a HAVING operator!

```
Find all jobs where the
average salary of employees
earning over 55000
is < 70000
```

```
SELECT Job
FROM Payroll
WHERE Salary > 55000
GROUP BY Job
HAVING avg(Salary)<70000;</pre>
```

,			
UserID	Name	Job	Salary
123	Jack	TA	50000
345	Allison	TA	60000
567	Magda	Prof	90000
789	Dan	Prof	100000

Pavroll
GroupBy-Aggregate

No need for a HAVING operator!			П _{Job}			
Find all jobs where the average salary of employees earning over 55000 is < 70000		σ _{S<70000'} γJob,avg(Salary)→S σ _{Salary>55000}				
						SELECT Job
WHERE Salary > 55000 GROUP BY Job HAVING avg(Salary)<70000;		UserID	Name	Job	Salary	
		123	Jack	TA	50000	
		345	Allison	ТА	60000	
		567	Magda	Prof	90000	

100000

Prof

Dan

789

GroupBy-Aggregate



GroupBy-Aggregate



The Greek alphabet soup:

- $\bullet\,\sigma,\Pi,\delta,\gamma$
- They are standard RA symbols, get used to them

Next: converting nested SQL queries to RA

Nested SQL to RA

Nested Queries to RA

RA is an algebra: has no nested expressions

 $\hfill\blacksquare$ We cannot write EXISTS or NOT EXISTS in σ

First unnest SQL query, then convert to RA

WITH Cardrivers AS
 (SELECT DISTINCT P.*
 FROM Payroll P, Regist R
 WHERE P.UserId=R.UserID)
 SELECT avg(Salary)
 FROM Cardrivers;







SELECT P.UserID, P.Name
FROM Payroll P
WHERE exists
 (SELECT *
 FROM Regist R
 WHERE P.UserID = R.UserID);

SELECT P.UserID, P.Name
FROM Payroll P
WHERE exists
 (SELECT *
 FROM Regist R
 WHERE P.UserID = R.UserID);

First unnest

```
SELECT DISTINCT P.UserID, P.Name
FROM Payroll P, Regist R
WHERE P.UserID = R.UserID;
```





SELECT P.UserID
FROM Payroll P
WHERE not exists
 (SELECT *
 FROM Regist R
 WHERE P.UserID = R.UserID);







```
SELECT P.UserID
FROM Payroll P
WHERE not exists
  (SELECT *
    FROM Regist R
    WHERE P.UserID = R.UserID);
```





```
SELECT P.UserID
FROM Payroll P
WHERE not exists
  (SELECT *
    FROM Regist R
    WHERE P.UserID = R.UserID);
```





Then unnest using set difference





Discussion

- SQL = declarative language; what we want RA = an algebra; how to get it
- We write in SQL, optimizers generates RA
- Some language resemble RA more than SQL, e.g. Spark

Next topic: how to design a database from scratch

Database Design

Database Design

New application needs persistent database.

The database will persist for a long period of time.
 We need a good design from day 1.

- Incorporate feedback from many stakeholders
 - Programmers, business teams, analysts, data scientists, product managers, ...

The Database Design Process



The Database Design Process



The Database Design Process



RA and ER

ER Diagrams

Entity-Relationship (ER) Diagrams

A visual way to describe the schema of a database

 Language independent: may implement in SQL, or some other data model Application to track the lifetime of products

- Keep information about Products: name, price, …
- Who manufactures them? Company name, address, their workers, ...
- Who buys them? Customers with their names, ...

Product























Example: adding Attributes

Next, let's design their attributes





Person

Example: adding Attributes






Example: adding Attributes





Example: adding Attributes



Example: adding Attributes



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RA and ER



RA and ER



